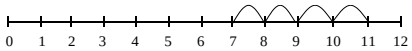
Stage 1: Addition (+)

Counting and Combining sets of Objects
Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)

Understanding of counting on with a number track.

Understanding of counting on with a number line (supported by models and images) e.g. 7+4



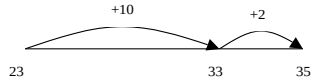
+ = signs and missing numbers
Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'. E.g. 2 = 1+ 1 or 2 + 3 = 4 + 1.

Missing numbers need to be placed in all possible places. 3 + 4 = □ ; □ = 3 + 4 ; 3 + □ = 7 ; 7 = □ + 4

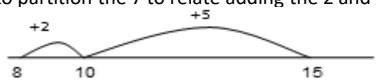
(to be extended and developed through all stages)

Stage 2: Addition (+)

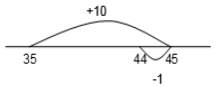
Counting on in tens and ones
23 + 12 = 23 + 10 + 2
= 33 + 2
= 35



Partitioning and bridging through 10.
The steps in addition often bridge through a multiple of 10
e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5. 8 + 7

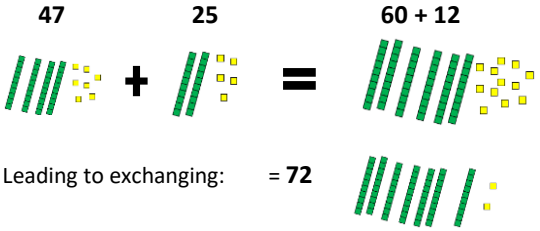


Adding 9 or 11 by adding 10 and adjusting by 1
e.g. Add 9 by adding 10 and adjusting by 1
35 + 9 = 44



Towards a Written Method
Partitioning in different ways and recombine

47+25



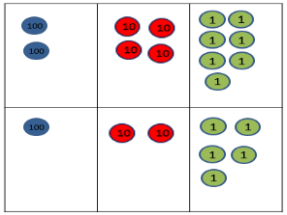
Expanded written method
40 + 7 + 20 + 5 =
40+20 + 7 + 5 =
60 + 12 = 72

40 + 7
+ 20 + 5
60 + 12 = 72

Stage 3: Addition (+)

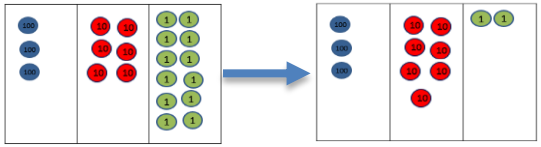
Partition into tens and ones
Partition both numbers and recombine.
Count on by partitioning the second number only e.g.
247 + 125 = 247 + 100 + 20+ 5
= 347 + 20 + 5
= 367 + 5
= 372
Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Towards a Written Method
Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)



200 + 40 + 7
100 + 20 + 5
300 + 60 +12 = 372

Followed by the exchange between tens and ones.

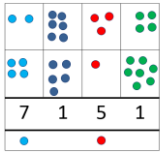


Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method as a more streamlined version of the expanded method, not a new method.

247
+125
372
10

Stage 4: Addition (+)

Compact written method
Extend to numbers with at least four digits.



2634
+4517
7151

Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

72.8
+ 54.6
127.4
1 1

Stage 5: Addition (+)

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency
e.g. 12462 + 2300 = 14762

Written methods (progressing to more than 4-digits)
Progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

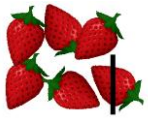
Stage 6: Addition (+)

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

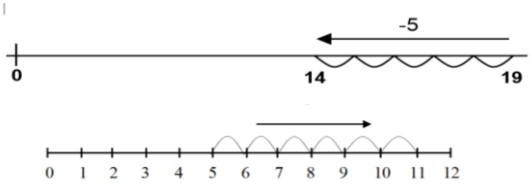
Written methods
Progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.
Continue calculating with decimals, including those with different numbers of decimal places

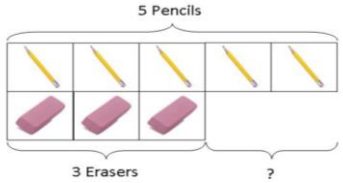
Stage 1: Subtraction (-)

Understand subtraction as take-away:



Understand subtraction as finding the difference:

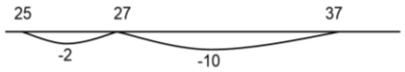


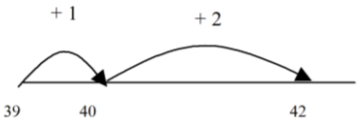


The model above would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.
The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings.

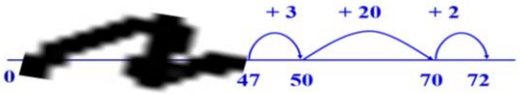
Stage 2: Subtraction (-)

It is valuable to use a range of representations (also see Stage 1).
Continue to use number lines to model take-away and difference.
E.g.

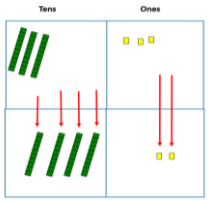




The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



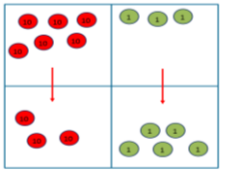
Towards written methods
Recording subtraction in expanded columns can support understanding of the quantity aspect of place value.The numbers may be represented with Dienes apparatus. E.g. 75 – 42



70 5
-40 2
30 3

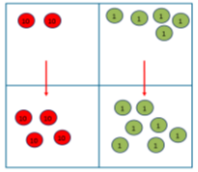
Stage 3: Subtraction (-)

Mental methods should continue to develop, supported by a range of models and images, including the number line. Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.
Written methods (progressing to 3-digits)
Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation)



90 8
- 30 5
60 3

For some children this will lead to exchanging, modelled using place value counters (or Dienes).

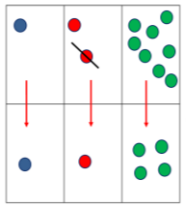


70 ¹2
- 40 7
20 5

A number line and expanded column method may be compared next to each other.

Stage 4: Subtraction (-)

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.
Written methods (progressing to 4-digits)
Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.



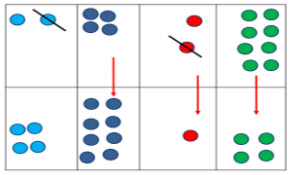
200 ²0 ¹2
- 100 10 4
100 10 8

Some children will move towards a compacted method:

²1
232
- 114
118

Stage 5: Subtraction (-)

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.
Written methods (progressing to more than 4-digits)
When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.



5 1 2 1
6232
- 4814
1418

Progress to calculating with decimals, including those with different numbers of decimal places.

Stage 6: Subtraction (-)

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods
As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured.

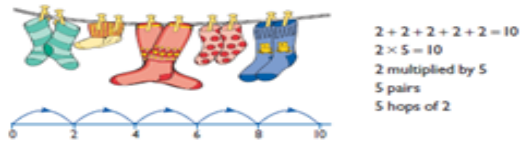
Move to examples of subtraction of decimal numbers of different decimal places including decomposition.

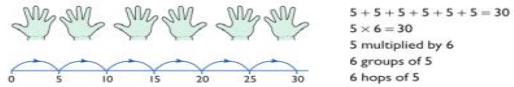
23.41
12.20
11.21

Stage 1: Multiplication (x)

Understand multiplication is related to doubling and combing groups of the same size (repeated addition)

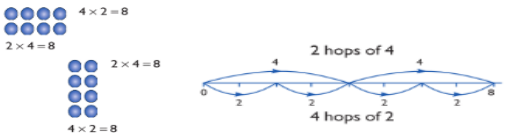
Washing line, and other practical resources for counting.
Concrete objects. Numicon; bundles of straws, bead strings





Use numicon, cuissenaire and bar method to develop the vocabulary relating to 'times' –
Pick up five, 4 times.

Use arrays to understand multiplication can be done in any order (commutative). Use objects to show this in a practical way .
2 groups of 4 objects or four groups of 2 objects.

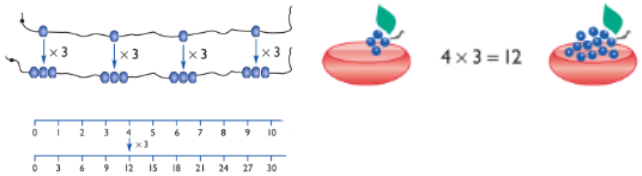


Stage 2: Multiplication (x)

Expressing multiplication as a number sentence using x
Using understanding of the inverse and practical resources to solve missing number problems.

7 x 2 = □ □ = 2 x 7 7 x □ = 14 14 = □ x 7
□ x 2 = 14 14 = 2 x □ □ x ◯ = 14 14 = □ x ◯

Develop understanding of multiplication using array and number lines (see Stage 1). Include multiplications not in the 2, 5 or 10 times tables.

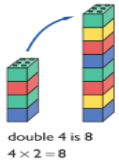


Doubling numbers up to 10 + 10: link with understanding scaling.
Using known doubles to work out the double of 2 digit numbers (double 15 = double 10 + double 5)

Towards written methods

Use jottings to develop an understanding of doubling two digit numbers.
e.g. 16 x 2 = 10 x 2 +6 x 2
(Introduction of grid method to model this)

Begin to develop understanding of multiplication as scaling (3 times bigger/taller)



Stage 3: Multiplication (x)

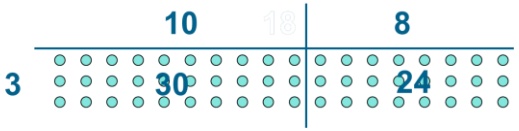
Mental methods
Doubling 2 digit numbers using partitioning

Demonstrating multiplication on a number line – jumping in larger groups of amounts

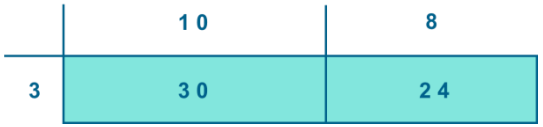
13 x 4 = 10 groups 4 = 3 groups of 4

Written methods (progressing to 2d x 1d)

Developing written methods using understanding of visual images



Develop onto the grid method

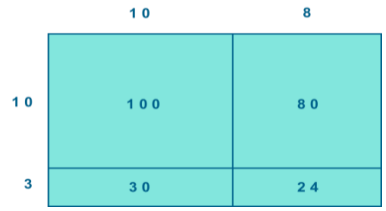


Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters.

Stage 4: Multiplication (x)

Mental methods
Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.
Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

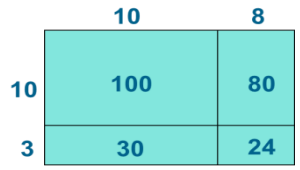
Written methods
Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays.

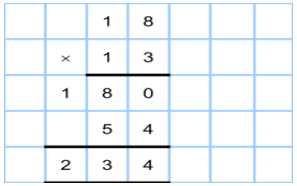


Stage 5: Multiplication (x)

Mental methods
X by 10, 100, 1000; Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35); Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning); Identify factor pairs for numbers.

Written methods (progressing to 4d x 2d)
Children to explore how the grid method supports an understanding of long multiplication (for 2d x 2d)

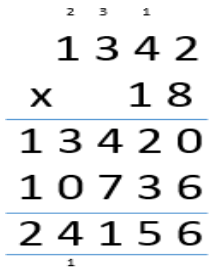




Stage 6: Multiplication (x)

Mental methods
Identifying common factors and multiples of given numbers
Solving practical problems where children need to scale up.
Relate to known number facts.

Written methods
Continue to refine and deepen understanding of written method using long multiplication.



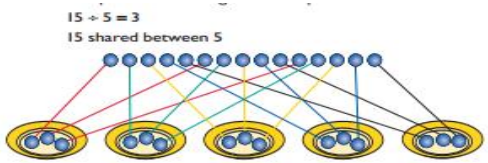
Stage 1: Division (÷)

Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.

Group AND **share small quantities- understanding the difference between the two concepts.**

Sharing

Develops importance of one-to-one correspondence.



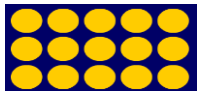
Children should be taught to share using concrete apparatus.

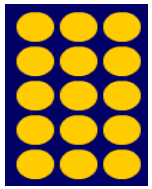
Grouping

Children should apply their counting skills to develop some understanding of grouping.

Use of arrays as a pictorial representation for division. 15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.

Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.





Stage 2: Division (÷)

÷ = signs and missing numbers

6 ÷ 2 = □ □ = 6 ÷ 2 6 ÷ □ = 3 3 = 6 ÷ □

□ ÷ 2 = 3 3 = □ ÷ 2 □ ÷ ∇ = 3 3 = □ ÷ ∇

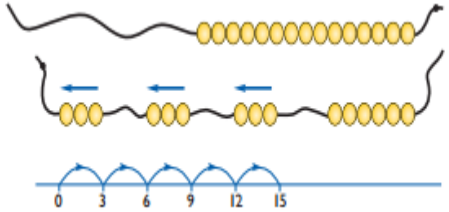
Know and understand sharing and grouping- introducing children to the ÷ sign.

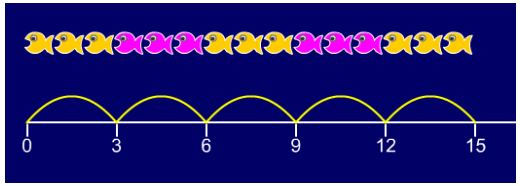
Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Grouping using a numberline

Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'.

15 ÷ 3 = 5





Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

Stage 3: Division (÷)

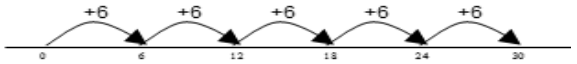
÷ = signs and missing numbers

Continue using a range of equations as in year 2 but with appropriate numbers.

Grouping

How many 6's are in 30?

30 ÷ 6 can be modelled as:



Becoming more efficient using a numberline

Children need to be able to partition the dividend in different ways.

48 ÷ **4 = 12**

+40 + 8

10 groups 2 groups

Remainders **49 ÷ 4 = 12 r1**

+40 + 8 + 1

10 groups 2 groups

Sharing – 49 shared between 4. How many left over?

Grouping – How many 4s make 49. How many are left over?

Place value counters can be used to support children apply their knowledge of grouping.

For example:

60 ÷ 10 = How many groups of 10 in 60?

600 ÷ 100 = How many groups of 100 in 600?

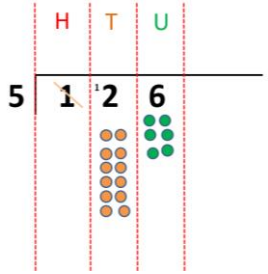
Stage 4: Division (÷)

Formal Written Methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number.

Short division to be modelled for understanding using place value counters as shown below.

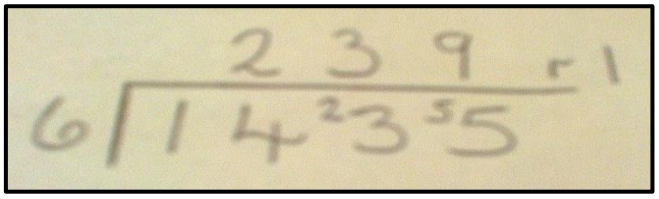
Calculations with 2 and 3-digit dividends.



Stage 5: Division (÷)

Formal Written Methods

Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used. E.g. 1435 ÷ 6

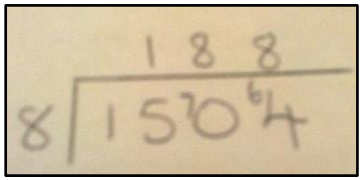


Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Stage 6: Division (÷)

Formal Written Methods – short division

E.g. 1504 ÷ 8



Extend to using jottings to support dividing by two digit numbers with bus-stop method.

E.g. 2364 ÷ 15

Quotients should be expressed as decimals and fractions